PRÁCTICA DIRIGIDA:

① La matriz $\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ ¿se encuentra en el espacio generado $\left\langle \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}; \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}; \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \right\rangle$

② $a, b, c \in V_3$; se verifica:

$a \times (b \times c) = (a \times b) \times c$

¿Los vectores $a \times b$; a y $b \times c$ son LI?

③ Dados los vectores $a = (1, m, m^2)$; $b = (1, e, e^2)$ y $c = (1, u, u^2)$

$\{a, b, c\}$, ¿es LI?

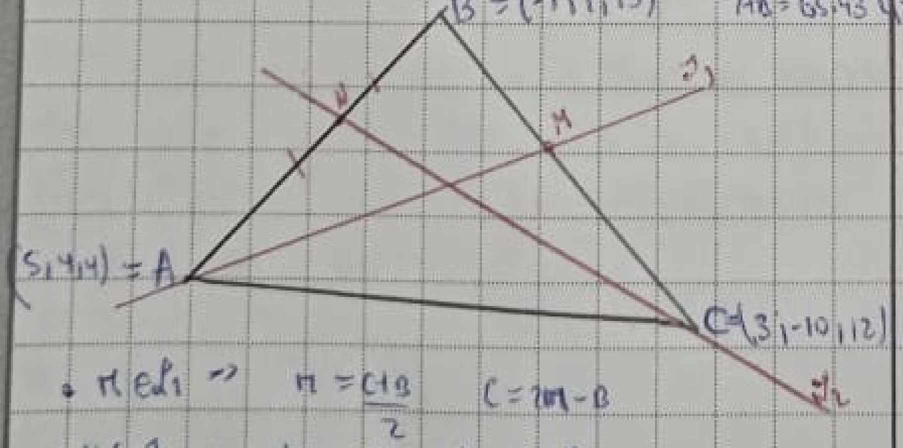
Sabemos que u, m, e son números diferentes.

④ Sea ~~ABC~~ ABC un triángulo, en sentido horario donde $B = (-1, 1, 13)$

$\mathcal{L}_1 : \dfrac{x+3}{-8} = \dfrac{y+13}{-17} = \dfrac{z-21}{17}$ es mediana relativa al lado $\overline{BC}$.

$\mathcal{L}_2 : \dfrac{x-1}{2} = \dfrac{y-15}{-25} = \dfrac{z-5}{7}$ es mediana relativa al lado $\overline{AB}$.

Determinar los vértices A y C.



• $M \in \mathcal{L}_1 \rightarrow M = \dfrac{C+B}{2}$ $\quad C = 2M - B$

• $N \in \mathcal{L}_2 \rightarrow N = \dfrac{A+B}{2}$ $\quad A = 2N - B$

• $C \in \mathcal{L}_2$: $C = P = 2M - B$ $\quad M \in \mathcal{L}_1$

• $A \in \mathcal{L}_1$: $A = P = 2N - B$ $\quad N \in \mathcal{L}_2$

(1) (r) $v = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ -1 & 1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -4 & 2 & 0 \\ 0 & 3 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & -4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -4 \end{bmatrix}$ $\langle A \rangle$ Si $\bar{a} \in \langle A \rangle \Rightarrow$

$a = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \alpha_4 v_4$

$\alpha_1 (1,2,0,0) + \alpha_2 (0,1,0,1) + \alpha_3 (2,0,2,0) + \alpha_4 (-1,1,1,-1) = (0,2,-1,0)$

$\alpha_1 + 0 + 2\alpha_3 - \alpha_4 = 0$
$2\alpha_1 + \alpha_2 + 0 + \alpha_4 = 2$
$0 + 0 + 2\alpha_3 + \alpha_4 = -1$
$0 + \alpha_2 + 0 - \alpha_4 = 0$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 2 & 1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -4 & 3 & 2 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -4 & 3 & 2 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 4 & -4 & -2 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -4 & 3 & 2 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & -6 & 0 \end{array}\right] \Rightarrow r(A) = r(A_a) = 4 = n = 4$

$\therefore \exists$ S. única

Si es combinación lineal

(2) $a \times (\underbrace{b \times c}_{D}) = (\underbrace{a \times b}_{E}) \times c$

$A = \{ a \times b;\ a,\ b \times c \}$ · $a = a \times b$ · $b \times c = c$

$A = \{ a, a; c \}$ LD $\vee$ $A = \{a \times b; a \times b; c\}$ LD

· $[abc] \neq 0 \Rightarrow [\underbrace{a \times b}_{u}\ \underbrace{a}_{v}\ \underbrace{b \times c}_{w}] = u \cdot (v \times w)$

$= (a \times b) \cdot [a \times (b \times c)] = (a \times b) \cdot (\underbrace{(a \times b) \times c}_{*})$

$= 0$

$*$ $(a \times b) \perp (a \times b) \times c$

$c \perp (a \times b) \times c$

Hε J_1: $H = \frac{B+C}{2}$

· $N \in J_2$ $N = \frac{A+B}{2}$

$(1,$

(A) :

$(-3, -13, 4)$

$\{$ ~~a+b+c~~) (abc) ≠ 0

$$\begin{vmatrix} 1 & m & m^2 \\ 1 & e & e^2 \\ 1 & u & u^2 \end{vmatrix} \sim \begin{vmatrix} 1 & m & m^2 \\ 0 & e-m & e^2-m^2 \\ 0 & u-m & u^2-m^2 \end{vmatrix} \sim \begin{vmatrix} 1 & m & m^2 \\ 0 & e-m & e^2-m^2 \\ 0 & u-e & u^2-e^2 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & m & m^2 \\ 0 & e-m & (e-m)(e+m) \\ 0 & u-e & (u-e)(u+e) \end{vmatrix} \sim (e-m)(u-e) \begin{vmatrix} 1 & m & m^2 \\ 0 & 1 & e+m \\ 0 & 1 & u+e \end{vmatrix}$$

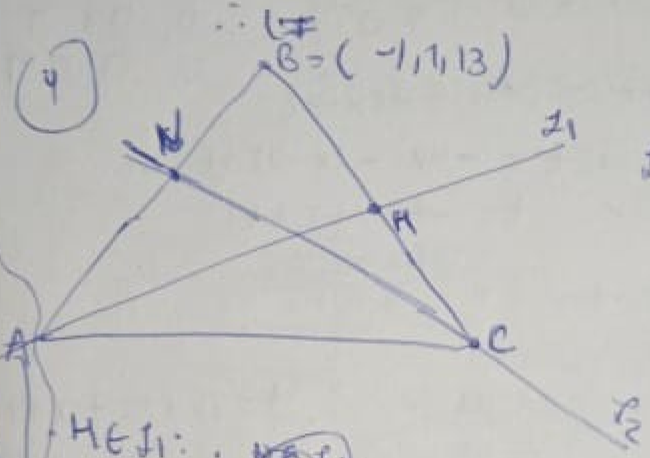
$$\sim (e-m)(u-e) \begin{vmatrix} 1 & m & m^2 \\ 0 & 1 & e+m \\ 0 & 0 & u-m \end{vmatrix} = (e-m)(u-e)(u-m) \neq 0$$

$e \neq m$ $u \neq e$

$u \neq m$

$\therefore e \neq m \neq u$

(4)



$\mathcal{L}_1$: $P_0 = (-3, -13, 21)$
$a = (-8, -17, 17)$

$\mathcal{L}_2$: $Q_0 = (1, 15, 5)$
$b = (2, -25, 7)$

$\cdot H \in \mathcal{L}_1$:

$H = \frac{B+C}{2} \Rightarrow C = 2H - B$

$(H \in \mathcal{L}_1 ; C \in \mathcal{L}_2)$

$\cdot N \in \mathcal{L}_2$

$N = \frac{A+B}{2} \Rightarrow A = 2N - B$ $(N \in \mathcal{L}_2 ; A \in \mathcal{L}_1)$

luego:

(C) : $C \in \mathcal{L}_2$

$(1,15,5) + r_1(2,-25,7) = 2[(-3,-13,21) + t_1(-8,-17,17)] - (-1,1,13)$ (α)

(A) : $A \in \mathcal{L}_1$:

$(-3,-13,21) + t_2(-8,-17,17) = 2[(1,15,5) + r_2(2,-25,7)] - (-1,1,13)$ (β)

$(\alpha = \beta)$